The Theory of Computation

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Solution Manual

Chapter 7, Additional Problem 7

The problem is clearly in NP. We prove it NP-complete by transforming *HP between specified points*. First we show a somewhat more general version to be NP-complete, where edge weights can have any positive value as well as -1. In that case, we simply give a weight of $|V|-3$ to all arcs out of s, and a weight of -1 to all other arcs. For an s–t path to have negative weight, it must have at least $|V|-2$ arcs of weight -1 to compensate for the arc of weight $|V|-3$, and thus a total of $|V|-1$ arcs, which makes it a directed Hamiltonian path between s and t. Now, with only weights of unit value allowed, we simply replace the arcs of weight $|V|-3$ by chains of $|V|-3$ arcs, each of weight 1. A simpler construction sets up a chain from s of just enough arcs to get the positive component, then has the branching. To allow even graphs of 3 or fewer vertices to be transformed, add matched arcs of weight +1 and -1, so that we need not worry about a negative number of arcs.