The Theory of Computation
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Solution Manual
Chapter 7, Additional Problem 4

Membership in NP is simple, but must be checked negatively, since there could be an exponential number of cycles; thus we remove the guessed set of vertices from the set of vertices and verify (by breadth- or depth-first search) that the induced graph is acyclic. We prove this problem NP-complete by transforming *VC* to it.

Given an instance $G = (V, E)$ and K of *VC*, we construct an instance $G' = (V', A)$ and K' of *FVS* by letting $V' = V$, $K' = K$, and $A = \{(u, v), (v, u) \mid \{u, v\} \in E\}$. Thus the resulting directed graph is identical to the original undirected one, except that each edge of the old graph is replaced by 2 arcs in the new graph, one in each direction. Notice that this establishes a cycle, which is basic to the proof. This construction can obviously be done in polynomial time.

The proof is now easy. By definition of a vertex cover, each edge of E is covered, hence each arc of A is covered; since any cycle must use at least one arc, each cycle passes through one vertex in of the cover. Conversely, since each edge of E gives rise to a cycle in A, a solution to *FVS* must include one of the two vertices in that cycle and thus cover each edge.