The Theory of Computation
Bernard M.E. Moret
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Solution Manual
Chapter 7, Additional Problem 7.30

We prove that all four versions are NP-hard by using them in a Turing reduction to solve *Hamiltonian Circuit*. We simply ask for the smallest disjoint cycle cover and count the number of cycles in it. Clearly, a Hamiltonian cycle covers the entire graph, so that the graph is Hamiltonian if and only if the smallest disjoint cycle cover has exactly one cycle in it. (The directed version of *HC* is known to be equivalent to the undirected version.)

To show that they are NP-easy, we use the standard search technique to discover the number of cycles in the optimal cover with a logarithmic number of calls, then proceed to build a cover of that size. Our query technique will simply be to merge two vertices joined by an edge and verify that the resulting graph still has a DCC of the same size. If the answer is yes, then the two vertices form part of a cycle in an optimal DCC, so we keep them merged and proceed; otherwise, they are in different cycles, so we remove their common edge and proceed. In either case, each query eliminates an edge of the original graph, so we obtain our answer with a linear number of queries. Details change somewhat among the four versions—as just outlined, the procedure applies directly to the undirected, vertex-disjoint version—, but the changes are rather minor.