The Theory of Computation
Bernard M.E. Moret
Addison-Wesley-Longman, 1998

Solution Manual
Chapter 7, Additional Problem 7.28

A graph is strongly connected if and only if there is a path between any 2 vertices. Equivalently, a graph is strongly connected if and only if there is a path from vertex 1 to vertex 2, from vertex 2 to vertex 3, . . . , from vertex $n-1$ to vertex n, and from vertex n to vertex 1. Since we can count to n in logarithmic space, the problem is clearly in NL: our machine guesses each path arc by arc; no path between the two vertices i and $i+1$ can be longer than $n-2$; we can verify the existence of each arc and their sequence in logarithmic space.

To show that the problem is complete, we transform *Digraph Reachability* to it. The new graph will have the same vertices as the old, all of the arcs of the old, plus an arc (n, i) for each $1 \le i < n$ and an arc $(i, 1)$ for each $1 < i \le n$. Thus, if n is reachable from 1, then there is a path from any i to any j of the form $i \to 1 \ldots n \to j$; conversely, there is a path from 1 to n only if n was reachable from 1 in the original problem, since our new arcs did not introduce any new path from 1 to n (something we can prove formally by showing that, if there exists a path from 1 to n in the transformed graph, then there exists one such path that does not use any of the new arcs). Hence n is reachable from 1 if and only if the new graph is strongly connected.