The Theory of Computation

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Solution Manual
Chapter 7, Additional Problem 16

Membership in NP is clear. We transform *Directed HC* to our problem. The transformation is almost simplistic; it relies on the fact that a graph that has a directed Hamiltonian circuit is a strongly connected graph (although the reverse may not be true). Given the graph $G = (V, A)$, our transformation produces the graph $G' = (V, \emptyset)$, the weight function $w\colon (v_i, v_j) \to 1$ if $(v_i, v_j) \in A$ and $w\colon (v_i, v_j) \to 2$ otherwise, and the bound $B = |V|$. The transformation can easily be carried out in polynomial time. Given a solution to *DHC*, we obtain a solution to our transformed instance by adding exactly those arcs used by the circuit. The total cost is B, since each such arc exists in A and thus has unit cost; the resulting graph is strongly connected since it has a directed Hamiltonian circuit. For the converse, observe that, in any strongly connected graph, each vertex must have degree at least 2; given the bound, each vertex will have degree 2 exactly, since at most $|V|$ arcs may be added. The only connected graph where every vertex has degree 2 is a circuit.