The Theory of Computation
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Solution Manual
Chapter 7, Additional Problem 14

Membership in NP is clear. We transform *Positive NAE3SAT* to our problem as follows. The number of partitions will be 2. For each clause in *NAE3SAT*, we place a triangle in our problem. For each variable, we set up a vertex in our problem. Finally, we connect the variable vertex to each of the triangle vertices corresponding to the occurrences of the variable in clauses by a connection piece which ensures that both ends of the connection must reside on the same side of any partition; that piece must also prevent any path from existing between its two ends on either side of the partition. Note that the connection piece that consists simply of a triangle, each vertex of which is connected to each of the two vertices (overall, a K_5 missing one edge), does not work, as it allows a path from one end to the other and thus may lead to cycles through two or more variable vertices and two or more clause triangles. A suitable piece builds upon this by connecting both endpoints to each end of a K_5 missing one edge (the edge between these two ends). The proof follows immediately.

From regular *NAE3SAT* we need an XOR piece rather than an equivalence piece; we can then use this XOR piece in a construction similar to our *MxC* construction. A suitable piece uses a K_4 in the middle, with the two end vertices connected to each other and each connected to the two vertices of the K_4 on its side. This piece forces the two endpoints on opposite sides, thereby also preventing any path through. (We can also get a piece from the failed triangle idea mentioned above: use two of these from one endpoint, leading to two new vertices; connect these two new vertices to the other endpoint; the other endpoint cannot be on the same side, or a cycle would exist through the equivalence components.)

An elegant construction from *G3C* simply adds a new triangle to the graph and connects every vertex of the original graph to each of the three vertices of the new triangle. Since the coloring induces no edge at all, we can also partition the triangle into 3 pieces and be done. In reverse, being by noting that the triangle must be cut in 3: otherwise, the one monochrome edge would close a cycle with any vertex of the graph that has the same "color". Thus the triangle provides one vertex in each of the 3 pieces; since each such vertex is connected to all vertices of the original graph, it follows that any edge in the original graph must broken lest a cycle occur.