The Theory of Computation
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Addison-Wesley-Longman, 1998

Solution Manual
Chapter 7, Problem 7.7

We solve the problem through direct self-reduction and also through reduction to the completion problem (given a partial TSP tour, find the optimal completion), which we prove to be NP-complete in its decision version.

The direct self-reduction is slightly tricky: once we have included a trip segment, say from city A to city B, we need to modify the problem (number of cities and distance measures) in order to enable us to use an optimal TSP for the modified problem to yield an optimal path through the remaining cities from B back to A. The obvious tool is merging all cities in the currently developed trip segment into a single city; the problem with this approach is how to define the cost of travelling from the merged city to the remaining cities—for instance, is the cost of travelling to C that of travelling from A to C or that of travelling from B to C? There are only two choices (we can only travel from the two ends of the developed trip segment), but they suffice to cause some trouble. The solution is to abandon symmetry (never required in the TSP problem in any case) and orient the currently developed trip segment, say from A to B, so that costs of travelling from the merged city to other cities are the costs of travelling from B to the other cities, while costs of travelling to the merged city are the costs of travelling to A. (Another, similar solution, is to remove all but the two endpoints of the developed trip segment and set up a cost of zero for travelling between the two endpoints of that segment.) Once this step is understood, the NP-easiness proof proceeds as usual: use binary search to obtain the optimal tour length, then start from some arbitrary city and develop the tour edge by edge by trying out in turn each possible edge from the last city reached, using the oracle to decide whether that edge can lead to an optimal solution.