The Theory of Computation

Bernard M.E. Moret

Addison-Wesley-Longman, 1998

Solution Manual
Chapter 7, Problem 7.56

This exercise does not really propose a problem—instead, it solves one. Its goal is to incite the reader to reflect on the construction described. Thus there is no additional proof to give here, but perhaps we can make a few comments on the construction.

It is perhaps surprising that the construction is so simple given how long we had to wait for a proof of NL = coNL. Note however that the construction is actually fairly involved: it uses recursion to build the set and nondeterminism in a somewhat devious way to count the number of solutions. It also checks its guesses through a clever mechanism. Finally, the proof of the theorem depends on yet one more realization, namely that counting can be parlayed into a negative answer by checking the number of distinct IDs. Thus, while the final result is very clear and elegant, there are good reasons for its having been rather slow in coming.

It is also immediately clear from the construction that the space bound is absolutely crucial; a time bound would not help here—there is no chance that we can somehow extend this technique to show something like NP = coNP, alas.