The Theory of Computation
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Solution Manual
Chapter 7, Problem 7.52

The answer is deceptively simple. Indeed, the problem as stated appears tricky only because of the deliberately vague language used. The key to the answer is to realize that classes and machines are not the same thing—we can typically attach a variety of different machines to the same complexity class. Indeed, we have done so repeatedly (in particular in Chapter 4) while convincing ourselves that the choice of machine ultimately did not matter.

The problems described in Exercises 6.25 and 7.51 are not problems about the classes NP or NP ∩ coNP; they are problems about classes of machines—more precisely, classes of pairs formed by a machine and a string. We could have P = NP = NP ∩ coNP and still have a completely different notion of a machine for P vs. a machine for NP ∩ coNP: even though both sets of machines would then have equivalent power, they would have different structure and descriptions—indeed, this is why we called one type semantic and the other syntactic: machines for syntactic classes can be described naturally with simple rules for their transitions (and can accept simply by entering an accepting state), while machines for semantic classes rely on complex descriptions of acceptance conditions.

Thus the problem is NP-complete for NP because we have a natural description of the NP machines, one that can be used in a simulation; but the problem is undecidable for NP ∩ coNP, even though we do not know whether the two classes are distinct, because the description of a machine for that class does not allow its direct simulation.