The Theory of Computation

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Solution Manual

Chapter 7, Problem 7.44

- at least 3 This is in NP: the certificate simply consists of 3 distinct truth assignments; it is easy to check that the three are distinct and that they are valid, both in polynomial time. It is in fact NP-complete: given an instance of *3SAT*, just add to it new variables and clauses that can be satisfied in exactly 3 ways (for instance, variables x, y, z and clauses $x + y + z$, $x + y + \overline{z}$, $x + \overline{y} + \overline{z}$, $\overline{x} + y + \overline{z}$, and $\overline{x} + \overline{y} + \overline{z}$); then the transformed instances has 3 times as many satisfying truth assignments as the original. Note that this can be done for any fixed value of k: i.e., if k is a constant and not a parameter, then "at least k" is NP-complete.
- at most 3 This is coNP-complete, as it is the complement of "at least 4."
- exactly 3 This is the combination of "at least 3" and "at most 3;" in other words, it is the combination of an NP-complete question and a coNP-complete question. As a result, it is certainly in Δ_2 (i.e., NP-easy); but we can do a bit better. Note that it is not necessarily in NP $\cup$ coNP, since it needs the power of both at once. We mentioned the class D_p, a superset of NP $\cup$ coNP and a subset of Δ_2, composed of languages that are intersections of an NP-complete and a coNP-complete language. This is exactly what our problem is and thus it is in D_p; it also turns out to be complete for D_p, but that is a bit harder to show.