The Theory of Computation

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Solution Manual
Chapter 7, Problem 7.38

The problem is in NL: since the operation is associative, w can be derived from S, if at all, as a chain of operations on the elements of S. Thus we can guess the chain and verify it operation by operation; all we need memorize is the result of the last operation verified (i.e., one element of A).

To show that the problem is complete, we transform *Digraph Reachability* to it. Let the digraph be given by $D = (V, A)$; we let the set of elements of our generation problem be $(V \times V) \cup \{(0, 0)\}$. The operation on our set is defined by $(i, j) \odot (k, l) \rightarrow (i, k)$ if $j = k$ and (0, 0) otherwise; this is clearly associative. Finally, we let $w = (1, n)$ and $S = \{(i, j) | (i, j) \in A\}$. It is obvious that w can be derived in our new problem if and only if n is reachable from 1 in the reachability problem. (Many other ways to define the operation; it always needs an absorbing element, however.)