The Theory of Computation
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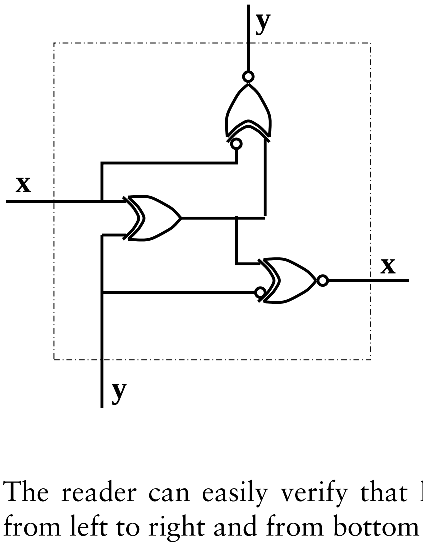
Solution Manual
Chapter 7, Problem 7.36

The problem is in P: *CV* itself is in P, so we need only verify planarity of the circuit's graph, which can be done in linear time as mentioned in Chapter 2.

To prove that the problem is complete, we transform the version of *CV* produced by our reduction from *PSA* to it. In a case like this, the standard approach is to develop a *crossing* gadget (see Chapter 8), that is, a planar circuit piece of polynomial size that can replace each crossing in an embedding of the (presumably) nonplanar original instance. The only crossings that can occur in our instance are crossings between a left-to-right propagation line and a vertical propagation line. So we need a planar circuit piece that will propagate these values properly; from the statement of the problem, it appears that negation will be required to realize such a circuit.

We use the fact that the XOR of two values can be decoded back into these values if we know one or the other value. We produce the following simple planar circuit.



The reader can easily verify that logical values are properly propagated from left to right and from bottom to top; inverting top and bottom yields a circuit that properly propagates logical values from left to right and from top to bottom. Thus we have a suitable crossing component.

Our circuit uses negation, as expected; unfortunately, it also uses XOR gates, whose direct implementation with AND and OR gates is not planar. The question was not entirely precise, since the *CV* problem could obviously be defined with any family of Boolean connectives. Our construction shows that *Planar CV* with connectives XOR, NOT, AND, and OR is P-complete. It remains to decide whether we can strengthen this statement by eliminating XOR gates from the choice of Boolean connectives.