The Theory of Computation

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Solution Manual

Chapter 7, Problem 7.31

We prove this problem NP-complete (membership in NP is obvious) by transforming *Hamiltonian Path* (HP) to it. We simply set up a matrix with $|E|$ columns and $|V|$ rows; each column corresponds to an edge and each row to a vertex; the entries are zeros and ones, where a_{ij} is one if and only if the ith vertex is an endpoint of the jth edge. Thus each column has exactly two one entries and $|V|-2$ zero entries. We then set K to $|V|-1$: the selected columns will correspond to the edges of the Hamiltonian path.

If the graph has a Hamiltonian path, we select those columns of the matrix that correspond to the edges of the Hamiltonian path. There are $|V|-1$ such edges, so we exactly meet the bound K.

Since each column has exactly two ones, making them consecutive by a permutation of the rows simply indicates that the edge corresponding to the column is between two vertices adjacent in the new row ordering. Since all edges are distinct and since we have $|V|-1$ of them, all adjacencies $(i, i+1)$ for $1 \leq i < |V|$ must be covered. But a Hamiltonian Path is exactly a permutation of the vertices (read rows) such that there is an edge between any two consecutive vertices. Thus our transformed instance has a solution if and only if the original problem does.