The Theory of Computation

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Solution Manual
Chapter 7, Problem 7.30

The problem is clearly in NP: we can verify in polynomial time (cubic time at worst) that no triangle exists on either side of the partition induced by the edge coloring. (For instance, simply create two graphs, each on the same set of vertices, one with the blue edges, the other with the red edges; then check each graph for the presence of a triangle.)

We transform *Positive NAE3SAT* to *Monochromatic Edge Triangle*. Our old idea of using a triangle for each clause works perfectly here, since each triangle will have to be partitioned in order to avoid forming a monochromatic one. The only problem left, therefore, is consistency. Consistency here is a bit more difficult than in the vertex version: we shall need a gadget that connects two edges in such a way that the two edges must always be colored the same way in any solution. Fortunately, our experience with K_5 in the previous problem helps. Note that the smallest graph that cannot be edge-partitioned without leaving a monochromatic triangle is not K_5: if we draw K_5 as a pentagon with all edges internal to the pentagon, we can color the pentagon edges one color and the others another and avoid any monochromatic triangle. K_6 cannot be partitioned so as to avoid a monochromatic triangle (easily verified), but K_6 minus an edge is not the smallest critical graph in terms of edges (it has 20 edges and the piece described below has only 19)—nor is it colorable in such a way that two designated edges always get assigned the same color. A suitable gadget can be derived much the same way we derived the gadget for the vertex version: we shall place a new triangle between the two edges and connect each endpoint of each edge to all three summits of the triangle; in addition, we also match the endpoints of the two edges. The result is shown below. As in the vertex version, we can now simply connect all clause triangle edges corresponding to the same variable into a simple chain by using our gadget for each link.

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