The Theory of Computation

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Solution Manual
Chapter 7, Problem 7.3

There are many ways to show this. That the problem is in NP is obvious. One reduction is to take the text reduction for *HC* and produce, not a cycle, but a path, with the same construction (just open the loop at one of the vertices of degree 2). A direct reduction from *HC* is also possible: given an arbitrary instance of *HC*, add 3 vertices, call them u_1, u_2, and v; u_1 is a vertex of degree 1 connected only to u_2, so that it must be at one end of the Hamiltonian path; v will also be a vertex of degree 1 and thus the other end of the path. Now, let x be some arbitrarily chosen vertex of the instance of *HC*; we connect u_2 to all the neighbors of x and connect v to x. For there to be a Hamiltonian path, we must start at v, move to x (no choice), visit all vertices of the original graph, then take an edge to u_2 and end at u_1. This is possible if an only if the original graph has a Hamiltonian circuit: if it does, we use it from x all the way to the last vertex and, instead or returning to x, move to u_2; if we have a Hamiltonian path, it has to be as described, which implies that the piece of it that moves from x to the predecessor of u_2 visits every vertex of the original graph and we can then return to x by construction.