# The Theory of Computation

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## *Solution Manual*
## *Chapter 7, Problem 7.28*

Membership in NP is trivial: bipartiteness can be tested in linear time by depth-first search. Notice that a graph is bipartite if and only if there exists a partition of its vertices into two subsets such that each edge of the graph is cut (has its endpoints on opposite sides of the partition). This suggests a very simple transformation from *MxC*. Given an instance of *MxC*, $G = (V, E)$ and $K$, we produce an instance of our problem by using the same graph but setting the bound to $|E| - K$. If the resulting graph is bipartite, all its edges can be cut, and its has $|E| - (|E| - K) = K$ of them left, thereby satisfying the cut requirement. Conversely, if the graph can be partitioned such that $K$ edges are cut, then removing the remaining uncut edges ($|E| - K$ of them) will make the graph bipartite.

- *Minimum vertex-deletion bipartite subgraph*

*Instance:* as above.
*Question:* as above, but vertices get deleted instead of edges.

Membership in NP is obvious. We transform *VC* to our problem as follows. Given an instance $G = (V, E)$ and $K$ of *VC*, we produce an instance $G' = (V', E')$ and $K'$ of our problem where $V' = V \cup E$, $E' = E \cup \{\{e, v_i\}, \{e, v_j\} | e = \{v_i, v_j\} \in E\}$, and $K' = K$. That is, we produce a triangle in $G'$ for each edge in $G$ (a construction used in the *Dominating set* example). The construction is easily carried out in polynomial time. Now, notice that each triangle thus formed will have to be broken in order to obtain a bipartite graph; that is, one vertex in each triangle must be removed. Thus, if we have a solution to *VC*, we obtain a solution to our problem by removing the covering vertices from the transformed graph: since the vertices form a cover, they include one endpoint of each edge and thus one point in each triangle. The graph left after removal of these $K$ vertices includes only edges of the form $\{e, v_i\}$, for $e = \{v_i, v_j\} \in E$, and is thus bipartite (just put the vertices in $V' - E$ on one side and the others on the other side). Conversely, a solution to our problem includes one point in each triangle; this point can be taken as a member of $V' - E$ without loss of generality (if the solution includes a degree 2 vertex from $E$, we can replace it by another vertex from the triangle, this time from $V' - E$, at no loss). Thus a set of $K$ vertices from $V$ includes one endpoint of each edge: it is a vertex cover. (The remark about vertices of degree 2 can be avoided by setting two extra vertices for each edge, with each vertex connected to both endpoints of the original edge; now both triangles must be destroyed, so that only original vertices can be used.)

Another transformation starts from *NAE3SAT* and just produces the usual triangle per clause, segment per variable, and connections between complements, with $K$ set to the number of clauses. The "don't care" literal in each clause is removed and the rest split: the proof is trivial.