The Theory of Computation
Bernard M.E. Moret
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Solution Manual
Chapter 7, Problem 7.21

(Vertex-) Dominating set (DS)

We prove this problem NP-complete by transforming *VC* to it.

Given an instance $G = (V, E)$, K of *VC*, we construct an instance $G' = (V', E')$, K' of *DS* by letting $K' = K$, $V' = V \cup E - \{v | v \in V$ and degree$(v) = 0\}$, and the new set of edges include all of the old edges, plus two new edges for each old edge, so that, in effect, we replace each edge in G by a triangle in G', with the new vertex being uniquely associated with the old edge. Notice that the triangle can be dominated by any of its three vertices. is obviously polynomial-time.

Now if we have a solution to *VC*, then each edge must be covered; such a cover is trivially a dominating set in G, since we just added the one vertex and two edges for each old edge and this new construction is dominated by either end-vertex. Conversely, a solution to the dominating set problem must include at least one vertex from each triangle (since the 2 new edges are the only edges emanating from the new vertex). If all such vertices are drawn from V, we are done, since there is at least one for each edge and at most K of them; if some of them are new vertices, we may replace them without damage by one of the old vertices. Thus we are done.

Another simple proof can be constructed directly from *3SAT*. For each variable, we set a pair of vertices (the 2 literals), connected by an edge; we also add two extra vertices for each such edge, each connected to the two literals (those are enforcers). For each clause, we set a vertex and connect it to the corresponding 3 literals. The bound will be set equal to the number of variables; a solution must be just a collection of literals corresponding to a truth assignment. The enforcers ensure that we must pick one of the two "literals" to be in the dominating set (as otherwise we'd have to pick the two enforcers and thus be unable to meet the bound); since we must pick one of each pair, we cannot pick any clause vertex; the correctness of the construction follows immediately.

(Edge-) Dominating set (DS)

Same as above, but we want to dominate the edges of the graph. An edge is dominated by vertex x if either x is an endpoint or x is connected to each endpoint by an edge. The proof proceeds easily with the same construction. If we further restrict domination by excluding the cover (thus now a vertex dominates an edge if and only if it is connected by an edge to both endpoints of the edge), much the same proof works, but now we replace each edge of VC by two adjacent triangles (really, a K_4 minus an edge) rather than just one triangle.