The Theory of Computation

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Solution Manual
Chapter 7, Problem 7.20

There is only one difference between *X3C* and *Three-Dimensional Matching (3DM)*: *3DM* introduces labeling and ordering—its elements come from three different sets of the same size and its 3-sets are really ordered triples (one element from each of the three sets). Membership in NP follows from the same considerations as for *X3C*. Completeness is proved by almost the same transformation. The change is in the ordering of the 3-sets and in the partition of the overall sets into three pieces.

Referring to pages 241 and 242, we will place all attaching points into one set, all even-indexed p_x^j (j even) into a second set, and all odd-indexed p_x^j (j odd) into the third set. The triples forming the variable pieces will be ordered, with the attaching point first, the even-indexed p_x^j second, and the odd-indexed p_x^j third. The 3-sets corresponding to a clause are also ordered, with the attaching point first, the literal second, and the truth indicator third; all literals are placed in the second set and all truth indicators in the third. For an instance of *Positive 1in3SAT* with n variables and m clauses, we get $6m$ attaching points and thus $6m$ elements in the first set; we then have $3m$ even-indexed and $3m$ odd-indexed p_x^j's, plus $3m$ literals and $3m$ truth indicators, so that each of the second and third sets also has $6m$ elements. The transformation then works without change.