The Theory of Computation

Bernard M.E. Moret

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Solution Manual

Chapter 7, Problem 7.18

We prove this problem NP-complete by transforming *Exact Cover By 3-Sets (X3C)* to it. The basic idea in the transformation is to add a fourth member to every triple of *X3C*, leaving enough choice that the necessary number of four-tuples can be selected—each with a different fourth member.

Given an instance S, $|S| = 3q$, $C \subseteq 2^S$, $(c \in C \Rightarrow |c| = 3)$ of *X3C*, we construct an instance T, $|T| = 4q$, $D \subseteq 2^T$, $(d \in D \Rightarrow |d| = 4)$ of *X4C* as follows: $T = S \cup \{x_1, \ldots, x_q\}$, $D = \{\{a, b, c, x_i\},\ 1 \leq i \leq q,\ \forall\{a, b, c\} \in C\}$. This construction can obviously be done in polynomial time.

The proof is simple. If *X3C* has a solution, then the q triples that form the solution determine a solution to *X4C*, where the i-th 4-tuple corresponds to the i-th triple augmented by x_i. The converse is equally simple: just drop the x_j from each 4-tuple in the solution to *X4C* to obtain a solution to the original *X3C*.