The Theory of Computation

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Solution Manual

Chapter 7, Problem 7.17

First, what might make the transformation work? Since a bipartite graph is simply a 2-colorable graph, one can take a 3-colored graph, remove all vertices of the least common color (there can be at most $\lfloor |V|/3 \rfloor$ of these), and thus obtain a 2-colored, i.e., bipartite graph. So this transformation is guaranteed to transform yes instances of *G3C* into yes instances of *MVDBS*. The trouble is that it can also transform no instances of the first into yes instances of the second. One can think of the problem as arising from the fact that one could delete vertices connected by an edge, whereas vertices of the same color cannot be so connected; in other words, there is nothing in the transformation to *force* the deletion of vertices from one color class. Another way to look at it is that the allowed bound is global, whereas bipartiteness is a very local property; hence one can take a graph with a lot of vertices, few edges, but a small, not 3-colorable subgraph (say K_4) and use most of the bound to remove the real problem vertices. For instance, K_4 plus two isolated vertices is not 3-colorable, yet gives rise to a bound of 2, which is sufficient to turn the graph into a bipartite graph by deleting 2 of the 4 vertices of the K_4 piece.