The Theory of Computation

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Solution Manual

Chapter 7, Problem 7.16

The answer is yes, provided that things are done right. We can go through the same imaginary construction, but then merge multiple edges into a single edge of corresponding weight. Since we want to cut two "lightweight" edges per clause and one per variable, we set the total weight of real (weighted) edges to be cut to $n + 2k$. The transformation is a bit more expensive than that in the text, but still easily done in low polynomial time. That a "yes" instance of *NAE3SAT* transforms into a "yes" instance of *Weighted MaxCut* follows immediately from our reasoning about *MaxCut* itself. The hard part is to show that "yes" instances of *Weighted MaxCut* can only result from "yes" instances of *NAE3SAT*. In order to proceed, begin by noting that the total weight of all edges in the instance is $n + 3k$, the number of "lightweight" edges. Now note that the n edges connecting literal vertices to their complements all have unit weights (because a literal and its complement cannot occur together in the same clause), so that they can contribute at most n to the total cut weight. From our construction, we cannot do better than cut a total of $2k$ "lightweight" edges from the edges set up for clauses, so that the maximum cut weight from clauses edges is $2k$. Thus we can conclude that all n truth-setting edges must be cut, thereby ensuring a legal truth assignment. To obtain a total cut weight of $2k$ from the clause edges, each of the k triangles we deliberately set up (there may well be others that were created unintentionally) must be cut: failure to cut any of them reduces the number of "lightweight" edges cut and thus the total cut weight. Thus the truth assignment is also satisfying.