The Theory of Computation

Bernard M.E. Moret

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Solution Manual
Chapter 7, Problem 7.15

If each edge is to be covered with exactly one vertex, we get a rather different problem. The text's transformation no longer suffices, since it produces instances where the optimal cover has edges covered at both ends (in the clause triangles). In fact, the problem is then in P and even solvable in linear time: basically, we are just asking if the graph is two-colorable in a certain way (with at most k vertices colored the same). Whenever one vertex is picked for the cover, the other endpoint must be discarded and any edge incident upon it must have its other endpoint in the cover. Thus the choice of a single vertex to place in the cover entirely determines what must be done with a connected component of the graph (in linear time); we can then count the number of vertices of each color and check whether either color has no more vertices than the given bound.