The Theory of Computation

Bernard M.E. Moret

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Solution Manual

Chapter 7, Problem 7.12

In our notation for the polynomial hierarchy, we want candidates for the class $\Sigma_2^p - \Delta_2^p$.

One potential source of such problems is inequivalence of languages, since the simplest way to prove inequivalence is surely to guess a string that belongs to one language but not to the other. For example, we saw that *Star-Free Regular Expression Inequivalence* is NP-complete. We need versions where the membership problem is harder to solve—basically where we need the power of Δ_2^p to solve it. Note, however, that we must still operate with a certificate of polynomial length; thus the languages cannot be too hard to decide; for instance, we saw that *Regular Expression Inequivalence* (where Kleene closure is allowed) is PSPACE-complete, a result that, under our standard conjectures, puts *REI* beyond PH. One example of a candidate for the class $\Sigma_2^p - \Delta_2^p$ is *Integer Expression Inequivalence* (which is in fact complete for Σ_2^p, for which see Exercise 7.50): the guess appears to require the full power of Δ_2^p for verification. We can turn to formal languages and consider inequivalence of various types of languages; while the details are beyond the scope of this exercise, it turns out that *Inequivalence of Context-Free Grammars on a One-Symbol Alphabet* (see Additional Exercise 7.29) is a good candidate. (It is a curious problem, since a context-free grammar on a one-symbol alphabet must in fact generate a regular languages; what is happening here is that we have taken a slightly too easy problem, *Inequivalence of Regular Grammars*, and turned it into a slightly harder one by increasing the expressive power of the description—context-free grammars can define some regular languages more concisely than a regular grammar could—, thereby providing one more example of the notion of "succinctness" introduced on pp. 218–219.

Another source of such problems is the *Satisfiability* family. We have already seen how to turn the basic NP-complete problem into a PSPACE-complete problem through arbitrary quantification of the variables. We can provide an excellent candidate for $\Sigma_2^p - \Delta_2^p$ by using bounded quantification instead. This problem is described in some detail on p. 271 of the text, so we will not repeat the description here, except to note that the problem is constructed exactly to the design we gave above: it requires a guess and that guess, while concise, requires in turn the full power of Δ_2^p for verification.