The Theory of Computation

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Solution Manual

Chapter 2, Problem 2.45

The argument used in Exercise 2.44 for polynomials in one variable can be used again (with a bit more dovetailing) to show that polynomials in two variables (or indeed in k variables for any fixed k) are countable. In fact polynomials in at most k variables (for any fixed k) are countable (just the sum of k countable sets). What, however, if we do not have a bound on k? It remains a countable set: we can dovetail once more, on all numbers of variables, to obtain the desired enumeration. This enumeration ends up dovetailing on all numbers of variables, on all degrees, and (for what is now a polynomial with a fixed number of terms) on all coefficient values for each coefficient.