The Theory of Computation
Bernard M.E. Moret
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Solution Manual
Chapter 2, Problem 2.43

Yes, such a set is countable. In fact, the rational numbers are such a set: think of the finite set as having two elements, call them n and d; then each function f from that set to $\mathbb{N}$ (minus the 0) defines the positive rational number $f(n)/f(d)$. The set of functions from S to T with S finite and T countable is, in effect, the set $T \times T \times \ldots \times T$, with $|S|$ copies of T in the Cartesian product; by dovetailing along $|S| - 1$ "dimensions" we will correctly enumerate that set.