The Theory of Computation
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Addison-Wesley-Longman, 1998

Solution Manual
Chapter 2, Problem 2.32

To obtain a derangement of the set $\{1, 2, \ldots, n\}$, we begin by mapping the first element, 1, to some other element of the set, say i, for which we have $n-1$ choices. The key at this point is to realize that it does not matter too much that we now have to map the remaining set $\{2, 3, \ldots, n\}$ to a different set of the same size, namely, $\{1, \ldots, i-1, i+1 \ldots, n\}$; the number of choices for derangements that we now have is easily related to the number of derangements of the set $\{2, 3, \ldots, n\}$, because any derangement of that set is a valid completion of our initial map, where we replace whatever was mapped to i (which, note, could not have been i) by a map to 1; in addition, we get the possibility of mapping i to 1 and any derangement of the set $\{2, \ldots, i-1, i+1, \ldots, n\}$. Thus our recurrence for the number of derangements of a set of size n is

$$d(n) = (n-1) \cdot (d(n-1) + d(n-2))$$

with $d(1) = 0$ and $d(2) = 1$. This is a very fast-growing function—it is clearly a double exponential, since the right-hand side is $n-1$ times the familiar Fibonacci right-hand side. This is also not a recurrence with a clean closed form, so we stop here.